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DATA REDUCTION OF AVERAGE FRICTION FACTOR OF GASEOUS FLOW IN MICRO-CHANNELS WITH ADIABATIC WALL

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KEY WORDS

Gas temperature, Fanno flow, Mach number, micro-tube.

ABSTRACT

This study focuses on data reduction of average friction factor of gaseous flow through micro-channels. In the case of microchannel gas flow at high speed, the large expansion occurs near the outlet and the pressure gradient along the length is not constant and increases near the outlet. This results in flow acceleration and a decease in bulk temperature. Therefore both pressure and temperature are required to obtain the friction factor of the microchannel gas flow. In the past data reduction of many experiments, the friction factors have been obtained under the assumption of isothermal flow since temperature measurement of compressible flow in micro-channels is quite difficult due to the experimental technique limitations. Kawashima and Asako [1] found that the gas temperature can be determined by the pressure under the assumption of one dimensional flow in an adiabatic channel (Fanno flow) to obtain the friction factor considering the effect of a decrease in gas temperature. They also reported the data reduction of the four multiplies of Fanning friction factor for the Fanno flow defined by [2]

$$f_{\mathrm{f}} = \frac{4\tau_{\mathrm{w}}}{\frac{1}{2}\rho u^2} = \frac{2D}{p}\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right) - \frac{2Dp}{\rho^2 u^2 RT}\left(\frac{\mathrm{d}p}{\mathrm{d}x}\right) - \frac{2D}{T}\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right) \tag{1}$$

To obtain a semi-local friction factor, pressures are measured at the pressure ports that are located very closely. In such a case, the change in temperature between two ports is small. Therefore, the temperature which appears in the second term in right hand side of Eq. (1) can be treated as constant. Integrating Eq. (1) with the average of the temperatures at two pressure ports, $T = \frac{T_1 + T_2}{2}$ where T_1 and T_2 are the temperatures at the pressure ports 1 and 2, we obtain

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$$f_{\mathrm{f}}^{*} = \frac{1}{x_2 - x_1}\int_{x_1}^{x_2} f_{\mathrm{f}}\,\mathrm{d}x = \frac{D}{x_2 - x_1}\left(-2\ln\frac{p_1}{p_2} + \frac{2\left(p_1^2 - p_2^2\right)}{R\left(T_1 + T_2\right)\dot{G}^2} + 2\ln\frac{T_1}{T_2}\right) \tag{2}$$

where $\dot{G}$ is mass flow rate per unit area.

However in the case that two locations are not close as inlet and outlet, T in the second term of Eq. (1) can not be considered as constant. It should be integrated between x_1 and x_2. Therefore, in the present study, the focus is to obtain the friction factor between two relatively distant points. The pressure loss determined by friction factors between two relatively distant points is also important to design piping lines. Fortunately, for an adiabatic channel flow, T at a location, x_{n} is determined by [1]

$$\alpha\frac{\rho_{\mathrm{in}}^2 u_{\mathrm{in}}^2 R^2}{2c_{\mathrm{p}} p^2} T^2 + T - \left(T_{\mathrm{in}} + \frac{u_{\mathrm{in}}^2}{2c_{\mathrm{p}}}\right) = 0 \tag{3}$$

and,

$$T = \frac{-1 + \sqrt{1 + 4\times\alpha\dfrac{\rho_{\mathrm{in}}^2 u_{\mathrm{in}}^2 R^2}{2c_{\mathrm{p}} p^2}\times\left(T_{\mathrm{in}} + \dfrac{u_{\mathrm{in}}^2}{2c_{\mathrm{p}}}\right)}}{2\times\alpha\dfrac{\rho_{\mathrm{in}}^2 u_{\mathrm{in}}^2 R^2}{2c_{\mathrm{p}} p^2}} \tag{4}$$

The above T (Eq. (4)) is a function of p. The average Fanning friction factor integrating Eq. (1) between x_{n} and $x_{\mathrm{n+1}}$ is

$$\begin{aligned} f_{\mathrm{f,ave}} &= \frac{1}{x_2 - x_1}\int_{x_1}^{x_2} f_{\mathrm{f}}\,\mathrm{d}x \\ &= \frac{D}{x_2 - x_1}\left(\begin{aligned} & -2\ln\frac{p_1}{p_2} + 2\ln\frac{T_1}{T_2} - \frac{1}{\left(\rho_{\mathrm{in}}^2 u_{\mathrm{in}}^2 R\times\left(T_{\mathrm{in}} + \dfrac{u_{\mathrm{in}}^2}{2c_{\mathrm{p}}}\right)\right)} \\ & \times\left(\frac{p_2^2 - p_1^2}{2} + \frac{B^2}{2}\ln\frac{p_2 + \sqrt{p_2^2 + B^2}}{p_1 + \sqrt{p_1^2 + B^2}} + \frac{1}{2}\left(p_2\sqrt{p_2^2 + B^2} - p_1\sqrt{p_1^2 + B^2}\right)\right) \end{aligned} \right) \end{aligned} \tag{5}$$

where

$$B^2 = 4\times\alpha\frac{\rho_{\mathrm{in}}^2 u_{\mathrm{in}}^2 R^2}{2c_{\mathrm{p}}}\times\left(T_{\mathrm{in}} + \frac{u_{\mathrm{in}}^2}{2c_{\mathrm{p}}}\right) \tag{6}$$

Therefore, the average Fanning friction factor between x_1 and x_2 can be obtained from Eq. (5)

Data reductions of the measured data for the stainless micro-tube (D=867 μm, L=200mm and Ra=0.448μm) in the previous study [3] were carried out to obtain the average Fanning friction factor between the inlet and the outlet, $f_{\mathrm{f,\,ave}}$ under the assumption of Fanno flow from Eq. (5). The outlet pressure of micro-tube is assumed to be atmospheric pressure. The obtained $f_{\mathrm{f,\,ave}}$ is plotted in Fig. 1 as a function of Reynolds number. The f_{f}^{*} obtained from Eq. (2) is also plotted in the figure. Mach numbers obtained using the measured local pressure at three locations near the micro-tube outlet

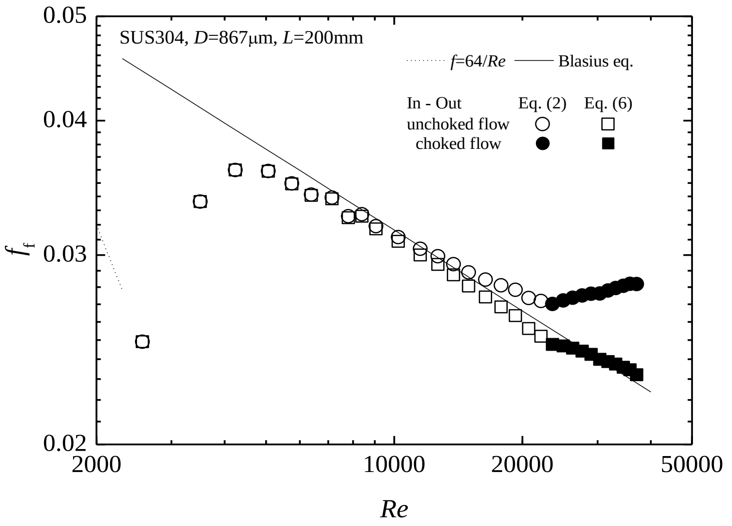

Figure 1: Fanning friction factor as a function of *Re* (*D*=867 μm).

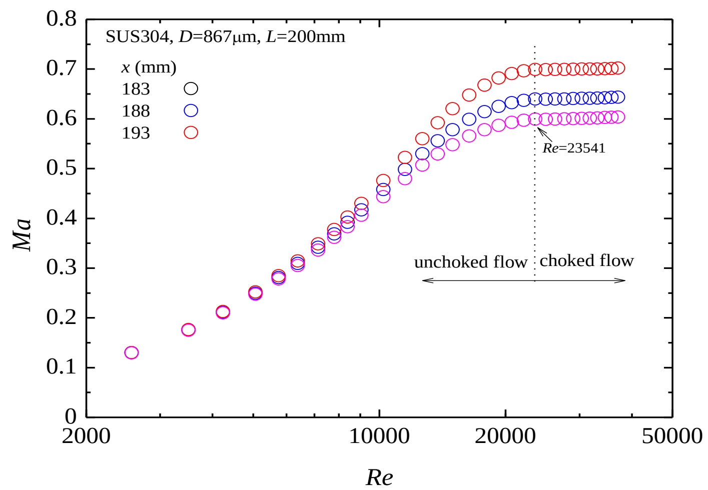

Figure 2: Mach number as a function of *Re* (*D*=867 μm).

are plotted in Fig. 2 as a function of Reynolds number. The values of Mach number increase with increasing Reynolds number and levels off in the range *Re* > 23000 since the flow is choked.

Due to the steep decrease in gas static temperature near the outlet by gas expansion, T in the second term of Eq. (1) can not be treated as constant. Therefore, the values of f_f^* obtained from Eq. (2) deviate from those of $f_{f,\,ave}$ obtained from Eq. (5) at *Re* > 10000. In the range of Re>23000, they deviate greatly from those of $f_{f,\,ave}$ obtained from Eq. (5) since the assumption of $p_{out} = p_{atm}$ is not valid with flow choking. When the flow choked the gas velocity and bulk temperature inside a tube remain unchanged and the outlet pressure is higher than the back pressure (atmospheric pressure) with the increases in Reynolds number. However, the outlet bulk temperature obtained under the assumption of $p_{out} = p_{atm}$ does not remain unchanged but decreases. Therefore the arithmetic average bulk temperature decreases and f_f^* increases at *Re* > 23000.

The values of $f_{f,\,ave}$ obtained from Eq. (5) are in excellent agreement with *Blasius* equation even though the outlet flow is under-expanded with flow choking and the outlet pressure is higher than the atmospheric pressure in the range of *Re* > 23000. The measure of under-expansion at the outlet is relatively small in this tube. In addition the relatively larger pressure difference between the inlet and the outlet is not significantly affected by under-expansion at the outlet.

References

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