MIGRATE2018:211909

COMPUTATIONAL STUDY OF THE LINEARIZED BOLTZMANN EQUATION FOR THE POISEUILLE, THERMAL CREEP AND COUETTE FLOWS FOR HARD SPHERE MOLECULES

Nikos Vasileiadis*, Dimitris Valougeorgis
Department of Mechanical Engineering, University of Thessaly, Pedion Areos, 38334, Volos, Greece
nikovasi@mie.uth.gr, diva@mie.uth.gr

KEY WORDS

Boltzmann equation, BGK, Discrete velocity method, hard sphere potential

ABSTRACT

The Linearized Boltzmann Equation (LBE) is the basic equation for tackling rarefied gas flows deduced by small local gradients of driving forces [1,2]. The LBE has been numerically solved by various computational methods [3-7]. However, even with today's computational power, the solution of the LBE remains computationally a very demanding task. To reduce the computational effort various kinetic models have been proposed by replacing the original collision operator with simplified expressions with the BGK kinetic model [8] being the most widely used one. In all kinetic models some approximation error is introduced, which may be determined by comparing the results provided by the kinetic model with the ones provided by the BE in some fundamental problems [3-7].

In the present work a source code to solve the LBE based on the discrete velocity method is developed. The code is validated and benchmarked by solving the classical Poiseuille, thermal creep and Couette flow problems and comparing the results with corresponding ones available in the literature [3-6]. Furthermore, a comparison with results obtained by the linearized BGK model is performed. In addition, a detailed investigation of the computational effort and accuracy for solving the LBE for different sets of velocity grids is provided. It is noted that the final objective is to apply the LBE code in rarefied gas mixtures flows and in order to achieve that, the present benchmarking task in the case of single gases is essential.

The investigated configuration concerns a gas between two infinite parallel plates set at $x' = \pm H/2$. In the Poiseuille and thermal creep flow configurations the gas is disturbed by a small pressure or temperature gradient in the y' direction respectively, while in the Couette flow the disturbance originates from the longitudinal movement of the two plates in opposite directions. The formulation of the BGK kinetic model has been thoroughly presented in [2] and is omitted here. Similar to the BGK or other kinetic models, the BE can be linearized by expressing the distribution function around the global Maxwellian distribution $f_0(\mathbf{c})$ through the so-called perturbed distribution function $h_i(x,\mathbf{c})$, with subscript $i = P,T,C$ denoting the Poiseuille, thermal creep and Couette problems respectively. Following the linearization procedure described in [6] it is shown that each of the perturbation functions $h_i(x,\mathbf{c})$ must satisfy:

* Corresponding author

$$c_x \frac{\partial h_i(x,\mathbf{c})}{\partial x} = 2\mu\delta\left[\int \frac{e^{-\mathbf{c}^2}}{\pi^{3/2}} K(\mathbf{c},\mathbf{c}^*) h_i(x,\mathbf{c}^*) d\mathbf{c}^* - \nu(\mathbf{c}) h_i(x,\mathbf{c})\right] + s_i(\mathbf{c}) \quad (1)$$

where $\mathbf{c}$ is the dimensionless molecular velocity, μ is the dimensionless viscosity, δ is the rarefaction parameter, K and ν are the dimensionless collision kernel and frequency respectively, while s_i corresponds to the source term of each problem:

$$s_P = -c_y, \quad s_T = -c_y\left(c^2 - 5/2\right), \quad s_C = 0 \quad (2)$$

The corresponding boundary conditions at each wall are:

$$h_i(-H/2,\boldsymbol{c}) = h_i^+(-H/2,\boldsymbol{c}) \quad c_x > 0, \quad h_i(H/2,\boldsymbol{c}) = h_i^+(H/2,\boldsymbol{c}) \quad c_x < 0 \quad (3)$$

where h_i^+ denotes the outgoing distributions. For a fully diffuse gas-surface interaction the outgoing distributions for the three different problems are:

$$h_P^+(\pm H/2,\boldsymbol{c}) = 0, \quad h_T^+(\pm H/2,\boldsymbol{c}) = 0, \quad h_C^+(\pm H/2,\boldsymbol{c}) = \pm c_y \quad (4)$$

It is noted that in both the BGK kinetic model and the LBE some velocity space variables can be eliminated in order to reduce the computational effort. In the BGK model the well-known projection procedure is used in order to eliminate the velocity components in the y' and z' directions, and thus the distribution function only depends on the spatial variable x and the magnitude of the c_x molecular velocity. In the LBE a velocity projection cannot be applied due to the form of the collision operator. However, by introducing polar coordinates for the velocity space the polar angle can be eliminated and thus the distribution function will only depend on the spatial variable x and the magnitudes of the c_x and c_r molecular velocities.

In Fig. 1 the dimensionless flow rate and heat flux are presented for the case of the pressure driven flow between parallel plates from the free molecular regime up to the transition regime. The results of the LBE show an excellent agreement with the ones provided in [5] with highest deviations of 0.1% and 0.5% for the flow rate and heat flux respectively. Comparing with the linearized BGK results it is seen that in the transition regime the flow rate results are in good agreement and the highest relative deviation is 4%. However, in the case of the heat flux the agreement is not good and deviations up to 20% are observed. This is contributed to the well-known fact that the BGK model provides an erroneous Prandtl number $\mathrm{Pr} = 2/3$ instead of the correct $\mathrm{Pr} = 1$. Similar results, for the dimensionless flow rate, heat flux and mean wall shear stress for the thermal creep and Couette flow configurations will be provided.

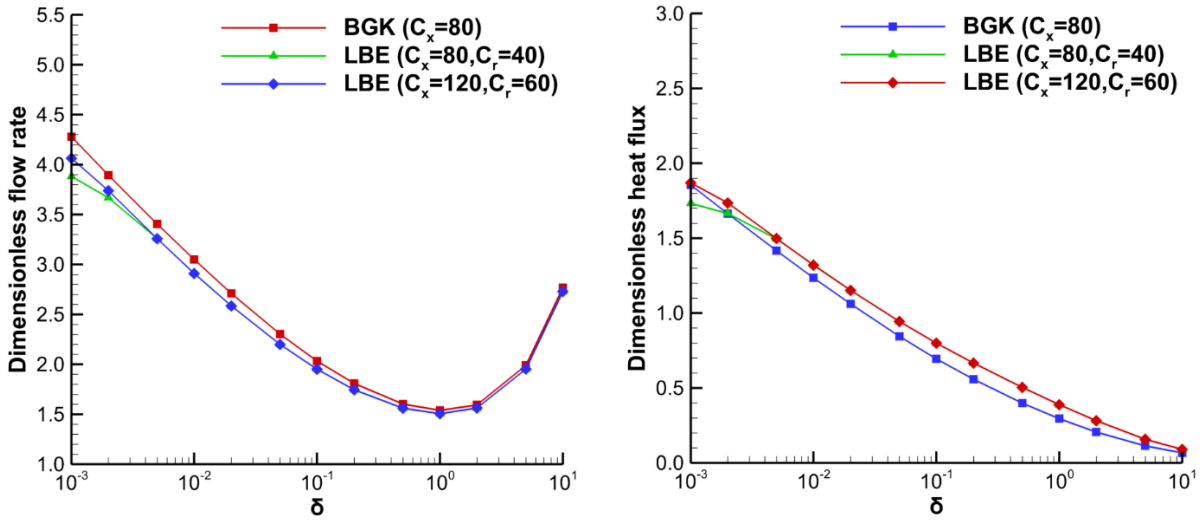

Figure 1: Dimensionless flow rate (left) and heat flux (right) against gas rarefaction in the Poiseuille problem.

As already mentioned, kinetic models circumvent the computation of the Boltzmann collision operator, thus significantly reducing the computational effort. In addition, the computation effort is further reduced since the solution of the BGK depends on two variables due to the applied projection instead of three variables required for the LBE. In Fig. 2 the CPU time required for the solution of the BGK and LBE with a convergence criterion of 10^{-6} for all macroscopic quantities is presented. It is observed that the CPU time required for LBE is around 4-5 orders of magnitude higher than the CPU time required for the BGK kinetic model. In addition, increasing the velocity grid from 80×40 velocities in the x and r directions to 100×80, leads to a computational time around 2.5 times higher. Furthermore, moving to a 120×60 velocity grid the computational time is increased by 4.5 times. It is noted, that in the BGK the dependence of computational time with respect to the velocity grid is linear. However, in the LBE the computational time has a polynomial dependence on the velocity grid with a power about 3.5-4 and this is contributed to the additional operations required for the computation of the collision kernel.

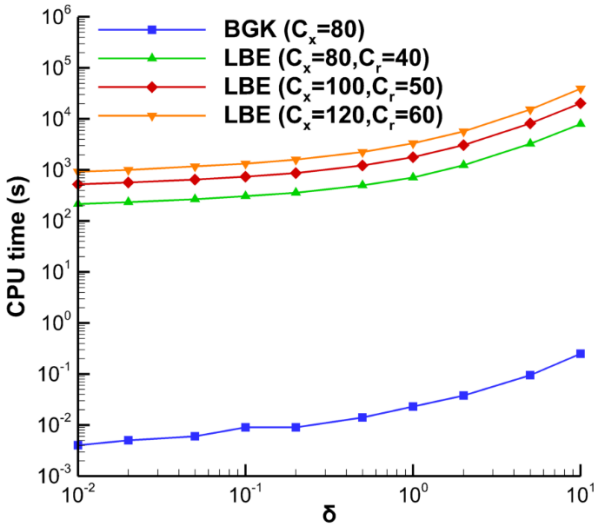

Figure 2: CPU time required for convergence of the Poiseuille solution.

The presented work is a preliminary investigation of the LBE in single gas flows before moving to the solution of rarefied binary gas mixture problems where kinetic models are not widely available, while the available ones, in most cases, have not been verified with respect to the Boltzmann equation.

Acknowledgements

This work has been supported by the European Community under the contract of Association EUROfusion-Hellenic Republic 2014-2018 (Grant Agreement No 633053, WP EDU). The views and opinions expressed herein do not necessarily reflect those of the European Commission.

Part of this work has been carried out by using computational time granted from the Greek Research & Technology Network (GRNET) in the National HPC facility ARIS under project ID005047.

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