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UNCERTAINTY PROPAGATION IN PRESSURE DRIVEN RAREFIED GAS FLOWS

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ABSTRACT

The propagation of the uncertainties of the input data to the computed mass flow rate is investigated in the case of pressure driven flow through a long tube. More specifically the uncertainties which are introduced in the measurements of the tube radius, the inlet/outlet pressures, the temperature of the gas and the accommodation coefficient are considered and the effect of each one on the computed mass flow rate is analyzed. Uncertainty quantification is important for the proper implementation of results [1]. The results obtained can provide an estimation of the uncertainty in many applications where such flows are encountered and show which parameter has a greater effect on the mass flow rate for various flow setups.

Kinetic formulation

The flow configuration consists of a long tube connecting two vessels kept at different pressures, at isothermal conditions. The tube radius, inlet and outlet pressure, temperature and accommodation coefficient are denoted as R, P_{in}, P_{out}, T and α respectively, while the tube length is L. Diffuse-specular Maxwell boundary conditions are assumed at the tube wall, with the accommodation coefficient $\alpha \in [0,1]$ denoting the percentage of particles undergoing diffuse reflection. Since only long tubes are considered, the infinite capillary theory can be applied and the pressure distribution and the mass flow rate can be computed using the following ordinary differential equation [2]:

$$\frac{dP(z)}{dz} = -\frac{\upsilon_0(z)}{\pi R^3 Q(\delta,\alpha)}\dot{m} \tag{1}$$

Here, z is the axial tube direction, $\upsilon_0(z) = \sqrt{2R_g T}$ is the most probably molecular speed with R_g denoting the gas constant and $\dot{m}$ is the mass flow rate. Also, the reduced flow rate Q is a function of the accommodation coefficient α and the rarefaction parameter $\delta = (PR)/(\mu\upsilon_0)$, where μ is the gas viscosity at some temperature T, and is taken from the solution of the linearized BGK model equation [2]. Given the inlet and outlet pressures, Eq.(1) is solved using a shooting method to provide the pressure distribution and mass flow rate.

Uncertainty propagation

The nominal values of the input quantities are denoted with the subscript n, while the uncertainty of some parameter x is denoted as $u(x)$. For example, the actual value of the tube radius is the sum of the nominal value plus the uncertainty, written as $R = R_n \pm u(R)$. The Monte Carlo Method (MCM) [3] is implemented for the uncertainty propagation. According to the MCM a large number of simulations are conducted. In each simulation, the value of some input quantity x with uncertainty $u(x)$ is sampled from a uniform distribution between $[x_n - u(x), x_n + u(x)]$. Upon conducting the simulations the probability distribution function of the output quantity, the mass flow rate in the present case, is constructed and the uncertainty of the output is defined as the 95% coverage interval.

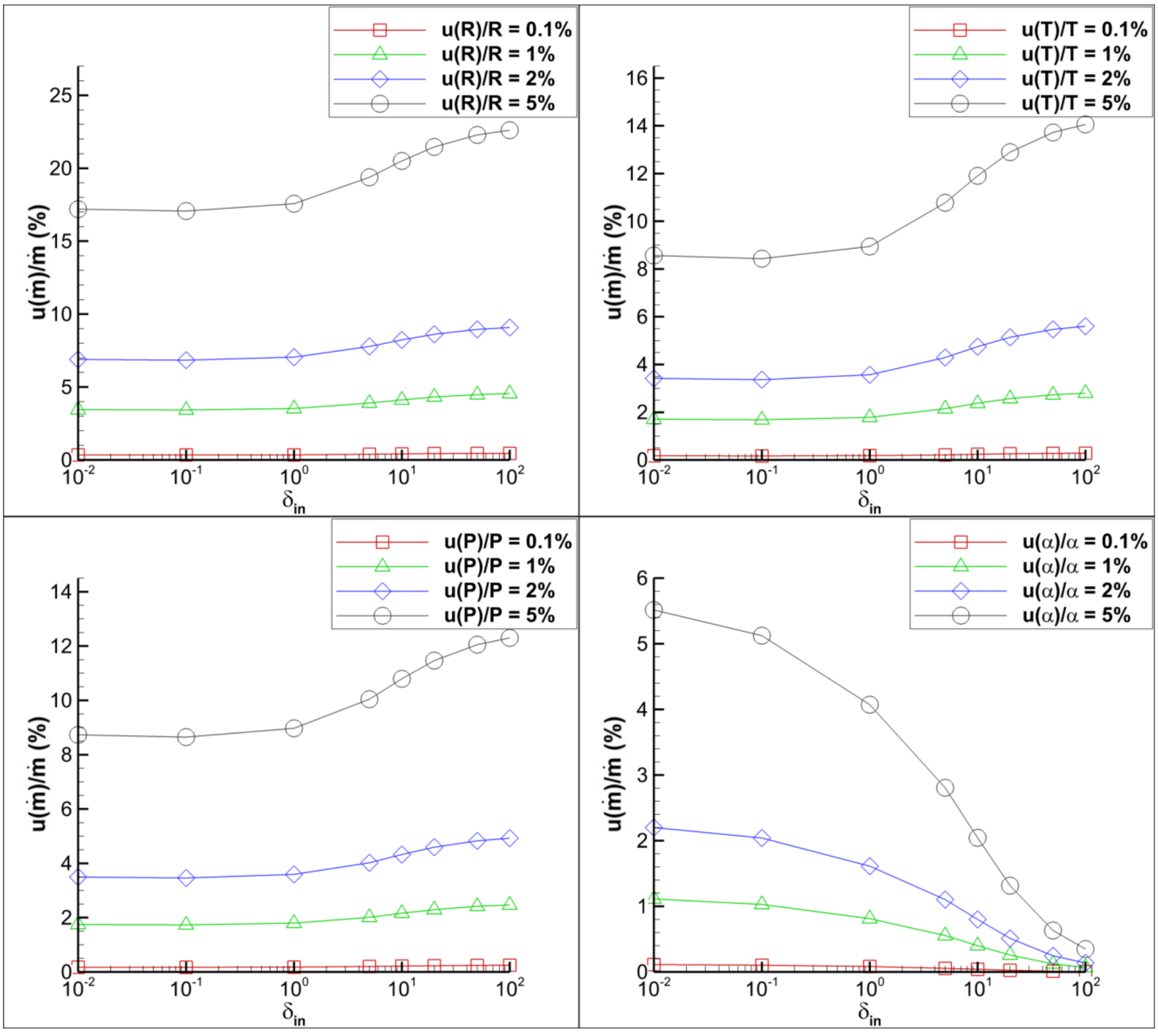

Figure 1: Relative uncertainty of the mass flow rate in terms of δ_{in} for various values of the relative uncertainty of each input parameter.

Results and Discussion

Simulations are conducted for various values of the inlet rarefaction parameter $\delta_{in} = (P_{in,n} R_n) / \left(\mu_n \sqrt{2 R_g T_n}\right)$ and the effect of the uncertainty of each parameter is analyzed individually by setting the uncertainties of all other parameters equal to zero. The flow parameters considered are $L / R_n = 20$, $P_{out,n} / P_{in,n} = 0.5$ and $\alpha_n = 1$. Results are presented in terms of relative uncertainty $\frac{u(x)}{x} \times 100\%$, and for each point 10^3 MCM trials are conducted.

Figure 1 shows the relative uncertainty of the mass flow rate in terms of δ_{in} ranging from the free molecular up to the continuum regimes for four values of the input relative uncertainty of each parameter, namely 0.1, 1, 2 and 5%. As expected, in all cases, larger input uncertainties lead to larger output uncertainties. The radius, temperature and pressure uncertainties are important for all values of δ_{in}, and have a slight increase at large values of δ_{in}. The accommodation coefficient uncertainty is important for small values of δ_{in} and its effect becomes negligible in the continuum regime.

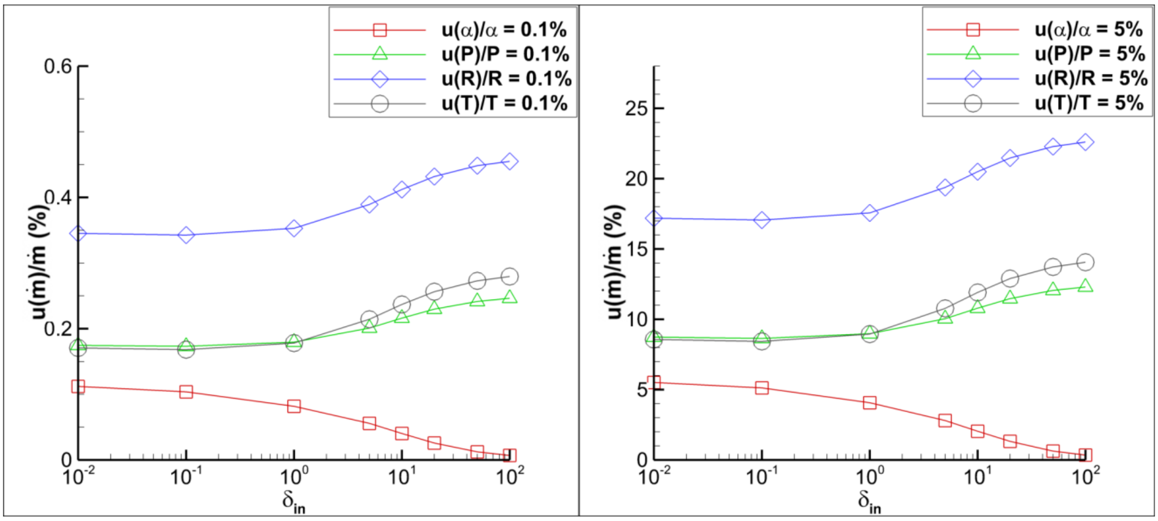

Figure 2: Relative uncertainty of mass flow rate in terms of δ_{in} for relative uncertainty of each input parameter equal to 0.1% (left) and 5% (right).

In Figure 2 a comparison of between the effect of the relative uncertainty of each input parameter on the relative uncertainty of the mass flow rate in terms of δ_{in} is shown. For both values of the input uncertainty (0.1% and 5%) the qualitative behavior of the output uncertainty is the same. In all flow regimes, the radius uncertainty is the most important one, the pressure and temperature uncertainties are tied and less important, while it is interesting to note that the accommodation coefficient uncertainty, when $\alpha_n = 1$, has a smallest effect compared to the other ones in all flow regimes.

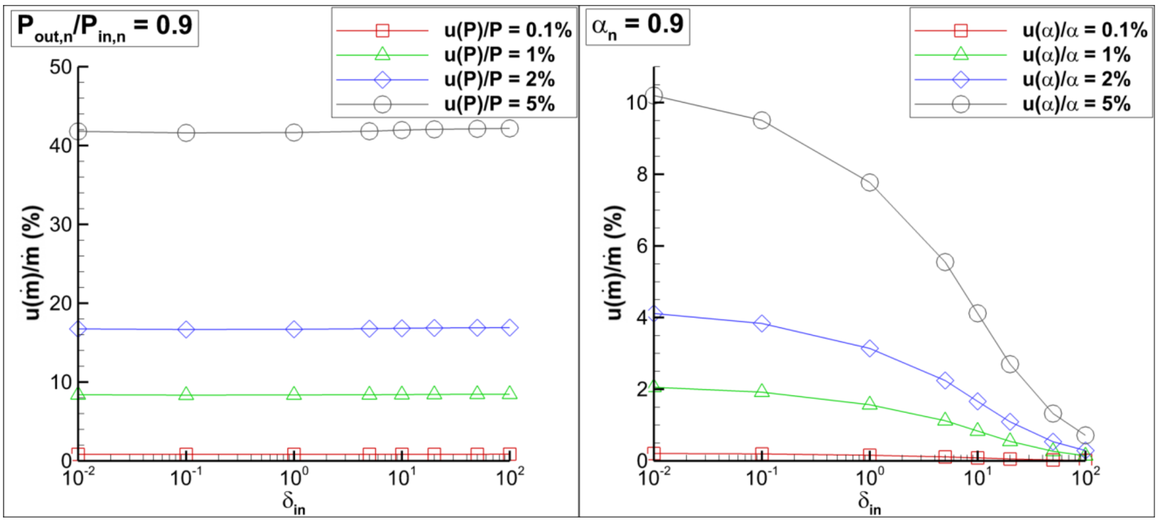

Figure 3: Relative uncertainty of mass flow rate in terms of δ_{in} for various values of $u(P)/P$ with $P_{out,n}/P_{in,n} = 0.9$ (left) and various values of $u(\alpha)/\alpha$ with $\alpha_n = 0.9$ (right).

Next, in Figure 3 corresponding results are obtained for a small pressure difference and diffuse-specular reflection. Figure 3 (left) shows that when the pressure difference is small (pressure ratio close to unity), the pressure uncertainties are magnified, compared to the respective values in Figure 1 for $P_{out,n} / P_{in,n} = 0.5$ and then, the pressure uncertainty (instead of the radius one) becomes the major factor of output uncertainty. Figure 3 (right) shows that when $\alpha_n = 0.9$ the output uncertainties are roughly doubled compared to the corresponding ones in Figure 1, where $\alpha_n = 1$. This is expected, as for the case of purely diffuse accommodation, where $\alpha = 1 \pm u(\alpha)$, the uncertainty is de facto decreased, since α cannot be larger than one.

In conclusion, the radius uncertainty is the most important in most cases, while the temperature and pressure uncertainties are of lesser importance, in the whole gas rarefaction regime. The accommodation coefficient uncertainty is important for highly rarefied flows, while its effect is diminished as the flow moves to the continuum regime. Also, the pressure uncertainty becomes the predominant factor of uncertainty for small pressure differences. The methodology for the uncertainty propagation implemented in the present work can be implemented in a straightforward manner in many rarefied gas flows and the deduced uncertainties on the overall quantities may aid researches and engineers engaged in the design of systems operating under rarefied conditions.

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References and Citations

[1] Alexeenko, A., Chigullapalli, S., Zeng, J., Guo, X., Kovacs, & A., Peroulis, D. (2011). Uncertainty in microscale gas damping: Implications on dynamics of capacitive MEMS switches. *Reliability Engineering and System Safety*, **96**, 1171-1183.

[2] Sharipov, F., & Seleznev, V. (1998). Data on internal rarefied gas flows. *J. Phys. Chem. Ref. Data*, **27**:657-706.

[3] BIPm, I., IFCC, I., ISO, I., & IUPAP, O. (2008). Evaluation of measurement data – Supplement 1 to the “Guide to the expression of uncertainty in measurement” – Propagation of distributions using a Monte Carlo method. Joint Committee for Guides in Metrology, JCGM 101:2008.